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BOTTOM HEAT TRANSFER TO WATER BODIES IN WINTER.(U)

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BOTTOM HEAT TRANSFER TO WATER BODIES IN WINTER

Kevin O'Neill and George D. Ashton

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In many surface water bodies, water temperature closely follows ambient air temperature. This means that warmer water in winter absorbs heat from below. The extent and pattern of winter heat gain is constrained by the fact that the water temperature does not fall below the freezing point. On the basis of a few simple assumptions, governing equations are solved here pertaining to heat flow in bottom sediments. The results are presented in general nondimensionalized curves. These allow estimation of water/sediment heat flux			

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20. Abstract (cont'd)

for any particular case, given truncation of the water temperature curve at the freezing point. The user must supply pertinent yearly air temperature mean and amplitude of variation, together with the thermal diffusivity for the bottom material. The governing equations are solved using a higher order finite element method which solves directly for temperature gradients and hence for heat flux. Thus the method provides particularly accurate flux values at high efficiency. The results illustrate in detail how winter water heat gain is less in cases where mean air temperatures are lower.

PREFACE

This report was prepared by Dr. Kevin O'Neill of the Geotechnical Research Branch, Experimental Engineering Division, and Dr. George Ashton of the Snow and Ice Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. The study was supported by the Office, Chief of Engineers, Directorate of Civil Works, under Work Unit 31594, Formation and Breakup of Reservoir Ice Cover and Effects on Thermal Energy Budget Computations. This work unit is part of the Environmental and Water Quality Operational Studies program managed by Jerome L. Mahloch of the U.S. Army Engineer Waterways Experiment Station.

The report was technically reviewed by Dr. Dennis Ford of WES.

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INTRODUCTION

One of the components of the total energy budget of a water body is the energy stored in and released from the bottom materials (usually sediments). The heat flux is seasonal, with heat conducted into the bottom during summer when the water is warm and released to the cold water during winter. This paper explores the magnitude of this heat flux, particularly in winter, when the water temperature is truncated at 0°C from the approximately sinusoidal variation that would otherwise persist.

NATURE OF THE PROBLEM

A reasonable representation of the annual temperature of a water body is a sinusoidal variation more or less following the average air temperatures. However, when the air temperature falls below 0°C the water temperature cannot follow it, because of the state condition; heat loss from water at 0°C to the subfreezing atmosphere manifests itself in ice formation while the liquid water remains at 0°C . In lakes with a complete ice cover, the cover also seals the water body from energy exchange with the atmosphere. Changes in the water temperature are dominated by heat flux from the bottom materials, which gradually raises the water temperature through the winter. While bottom heat fluxes in summer are generally not important components of the energy budget of water bodies, they are important during the winter.

A completely rigorous treatment of the bottom heat flux contribution requires specification of the water temperature at the sediment/water interface to include the gradual warming due to the heat flux. Nevertheless, a good approximation is to assume a constant bottom surface temperature of 0°C during subfreezing weather (Fig. 1). We also note that the constant 0°C water temperature during the winter is an excellent approximation for rivers and, while the associated heat flux causes only a small temperature rise (on the order of 0.01°C), the heat flux itself may have a significant effect on frazil ice in the flow. Finally, we note that the strictly sinusoidal surface temperature variation on a semi-infinite medium is a classic problem in conductive heat transfer and has been solved analytically¹.

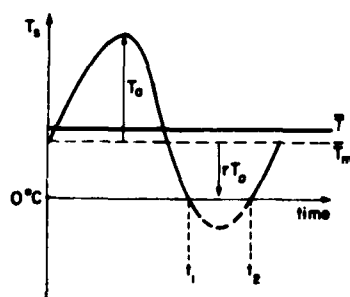


Figure 1. Generalized river bottom temperature variation with time. Between time t_1 and t_2 the ambient air temperature follows the dashed line, while the water temperature does not drop below 0°C .

¹Carslaw, H.S. and J.C. Jaeger (1973) Conduction of Heat in Solids. Second Edition. London: Oxford University Press, p. 64-70.

ANALYSIS

The specified surface temperature variation is shown in Figure 1. T_s is the sediment surface temperature at any time t , \bar{T}_m is the mean and T_a the amplitude of the strictly sinusoidal (air) surface temperature; \bar{T} is the mean of the truncated sinusoidal variation and is the temperature which will be approached at very great depths in the bottom after imposition of the truncated surface temperature over many years. These quantities ideally would be obtained from several years of water temperature measurements. As a practical matter, however, a reasonable approximation is to fit a sinusoidal variation to average monthly air temperatures obtained from nearby meteorological stations, since the water temperature variations of most shallow water bodies closely follow air temperature variations.

The governing heat conduction equation is

$$\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2} \quad (1)$$

where $T(x,t)$ is the temperature in the bottom at distance x below the sediment/water interface and $K = k/\rho c$ where k is the thermal conductivity of the bottom material, ρ is the density, and c is the heat capacity. (The Celsius temperature scale is assumed below. In principle, any other scale would result in identical expressions, as long as the freezing temperature of water is taken as zero degrees.)

The boundary conditions are

$$T(0,t) = \bar{T}_m + T_a \sin 2\pi ft, \quad t \text{ not between } t_1 \text{ and } t_2 \quad (2)$$

$$T(0,t) = 0 \quad t_1 \leq t \leq t_2 \quad (3)$$

$$T(\infty,t) = \bar{T} \quad (4)$$

$$T(x,0) = T. \quad (5)$$

In eq 2, f is the frequency, which we take to be 1 year^{-1} .

It is convenient to non-dimensionalize the equation by introducing

$$\theta = \frac{T - \bar{T}_m}{T_a} \text{ so at } x = 0, \quad 1 \leq \theta_s \leq -r \quad (6)$$

where $r = \bar{T}_m/T_a$. Also, using

$$\tau = ft, \quad \xi = x/\sqrt{K/f} \quad (7)$$

the equations assume the form

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \xi^2} \quad (8)$$

$$\theta(\infty, \tau) = \theta(\xi, 0) = \frac{\bar{T} - \bar{T}_m}{T_a} \quad (9)$$

and the boundary condition in eq 2 and 3 is scaled in the same manner.

Since we are primarily interested in the temperature gradient at the sediment/water interface, we seek to determine $\partial \theta / \partial \xi$ there as a function of r and time. At this point the advantages of nondimensionalization are apparent: all possible results may be displayed in a single set of curves in $\partial \theta / \partial \xi$ versus τ . Once $\partial \theta / \partial \xi$ is determined, the temperature gradient at the sediment/water interface is determined by the transformation

$$\frac{\partial T}{\partial x} = T_a \frac{\partial \theta}{\partial x} = T_a \frac{\partial \xi}{\partial x} \frac{\partial \theta}{\partial \xi} = \frac{T_a}{\sqrt{K/f}} \frac{\partial \theta}{\partial \xi} \quad (10)$$

The problem was analyzed using a finite element formulation described in more detail in the next section. The resulting output for the case of $r = 1$ (no truncation) was compared to the analytical solution and matched to three significant figures. When $r \neq 1$, the program was run for about five periods to remove transients. About 20-36 time steps per period (1 year) were used with a 10-element mesh (11 mesh points), which resulted in a very low cost per run. This efficiency was due to 1) a high order of interpolation between mesh points and 2) the particular implicit formulation in time.

THE FINITE ELEMENT ANALYSIS

In this section, a brief outline of the basics of the Galerkin finite element method is given. It is by no means sufficient for answering all questions concerning the method and the reader is referred to the voluminous literature for further information².

In the finite element method, mathematical functions are expressed as sums of finite series, utilizing a preselected set of so-called basis functions. Thus, if F is the function in question, it is expressed as

$$F = \sum_{j=1}^N F_j U_j \quad (11)$$

²E.g.: Zienkiewicz, O.C. (1977) The Finite Element Method. Third Edition. New York: McGraw-Hill.

where the F_j are coefficients, N is a finite integer, and the functions $U_j(x)$ are basis functions. In heat flow problems, the coefficients are usually time-dependent, varying according to the particulars of the problem, whereas the basis functions are written in space in a temporally invariant manner:

$$F = \sum_{j=1}^N F_j(t) U_j(x) . \quad (12)$$

The basis functions may be of many kinds, but in any case, are conceptually the first N members of a set which, in its entirety, could theoretically express the function F exactly. The use of such a series and the subsequent manipulations of it are reminiscent of the use of Fourier series in solving many problems. A major difference is that the sinusoidal functions in a Fourier series are in general nonzero throughout the entire domain. In contrast, basis functions for finite elements are typically locally defined, each being nonzero only within the space elements adjacent to some particular mesh point ("node") with which the basis function is associated (Fig. 2). An arbitrary function may be approximated using the functions depicted in Figure 2 by selecting appropriate coefficients, in this case the approximated function's values at the node points (Fig. 3).

The basis functions need not be linear, nor must the coefficients correspond to the function's values only, but may correspond, for example, to its derivatives.

Consider now a differential equation to be solved, written in the abstract as

$$L[y] = 0 \quad (13)$$

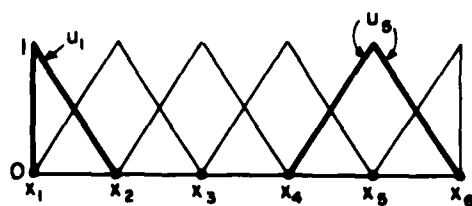


Figure 2. Typical linear finite element basis functions, on a uniform grid. U_5 , for example, rises to a magnitude of 1 at node point x_5 , declines to 0 at x_4 and x_6 , and is uniformly 0 everywhere except between x_4 and x_6 . The domain between any two node points constitutes one finite element.

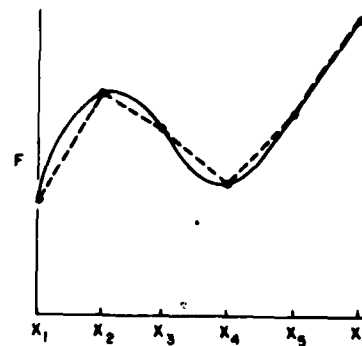


Figure 3. A function F , shown as the solid line, may be approximated in the manner of eq 12. Using the basis functions in Figure 2 with the function values of F at the nodes as the coefficients, one obtains the approximation shown as the dashed line.

where L is some differential operator. For example, if L corresponds to

$$L = \frac{\partial}{\partial \tau} - \frac{\partial^2}{\partial \xi^2}, \quad (14)$$

then eq 13 denotes

$$\frac{\partial y}{\partial \tau} - \frac{\partial^2 y}{\partial \xi^2} = 0. \quad (15)$$

The solution to the equation is some function y , for which the relation expressed in eq 13 holds exactly. If, instead of y , one substitutes into eq 13 an approximation of it, denoted \tilde{y} , then the equality will not be exactly satisfied, and

$$L[\tilde{y}] = R(x). \quad (16)$$

R is called the "residual," and inasmuch as \tilde{y} does not satisfy eq 13 exactly, R will not be uniformly zero.

It is the undertaking of a class of numerical methods called "weighted residual methods" to find approximations \tilde{y} which come desirably close to satisfying the governing equation. Specifically, it is required that

$$\int_S w_i R \, dx = \int_S w_i L[\tilde{y}] \, dx = 0 \quad (17)$$

where $w_i(x)$ is one of a selected set of weighting functions, and S is the domain considered. For example, if w_i is uniformly equal to one, then eq 17 requires only that the net or sum of the residual over the entire domain be zero (an unreliable criterion). The functions w_i may be any of a considerable variety of possibilities, each giving rise to a different specific method of attack, such as the collocation method, the subdomain method, and the least squares method.

In the Galerkin finite element method, the set of weighting functions is chosen to be the basis functions themselves. Both experience and theoretical analysis justify the expectation that the use of the U_i as weighting functions can produce accurate approximations to the desired y . Thus, one approximates y as

$$\tilde{y} = \sum_{j=1}^N Y_j(t) U_j(x) \quad (18)$$

where the Y_j are initially known, and in accordance with eq 17 one then writes

$$\int_S U_i L[\sum_j Y_j U_j] \, dx = 0 \quad (19)$$

or

$$\sum_{j=1}^N \left\{ \int_S U_j L[U_j] dx \right\} Y_j = 0. \quad (20)$$

One such equation results for each U_i in the set of N . In matrix notation, eq 20 is equivalent to

$$[A] \{y\} = 0. \quad (21)$$

When the boundary conditions are incorporated, eq 21 constitutes a set of N linear algebraic equations in the N coefficients sought, which can be solved by any appropriate standard method.

All results reported below were obtained using a particular set of basis functions known as Hermite basis functions. These functions have the property of generating a particularly accurate, higher order (cubic) interpolation of functions between the node points. They are also selected so that half the unknown coefficients correspond to function values at the nodes; the other coefficients correspond to the function's gradient values. Thus gradients of the unknown function at the node points are obtained directly in the course of solving the problem, without further manipulation or numerical differentiation. This system is especially desirable for the problem treated here, since flux values based on solution gradients are the result ultimately sought.

RESULTS AND INTERPRETATION

In Figure 4 are presented curves of $\partial\theta/\partial\xi$ at $x = 0$, for $r = 1.0$ (no truncation), 0.9, 0.8, 0.6, 0.4 and 0.0 (mean air temperature = 0°C). It is clear from Figure 4 that the truncation of the surface temperature has a significant effect on the temperature gradient, particularly during the winter.

Practical use of these results to calculate the bottom heat flux contribution requires knowledge of the surface temperature mean and amplitude, and the bottom thermal diffusivity K . For shallow water bodies it is reasonable to use mean monthly air temperature to determine the former. The thermal diffusivity depends upon the thermal properties of the bottom materials. However, the bottom materials of most water bodies are sediments saturated with water. Measurements of the thermal conductivity, from which the diffusivity may be calculated, show that bottom materials may be considered to have the same diffusivity as still water³. Accordingly, in the examples which follow, the diffusivity K will be taken as $1.31 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$, which is the value for water at 0°C .

As an example, we will use the mean air temperatures for Hanover, New Hampshire (Fig. 5). The mean annual temperature (T_m) is 6.3°C and the

³McGaw, R. (1974) Thermal conductivity of organic sediments from two Wisconsin lakes. U.S. Army Cold Regions Research and Engineering Laboratory Special Report 129.

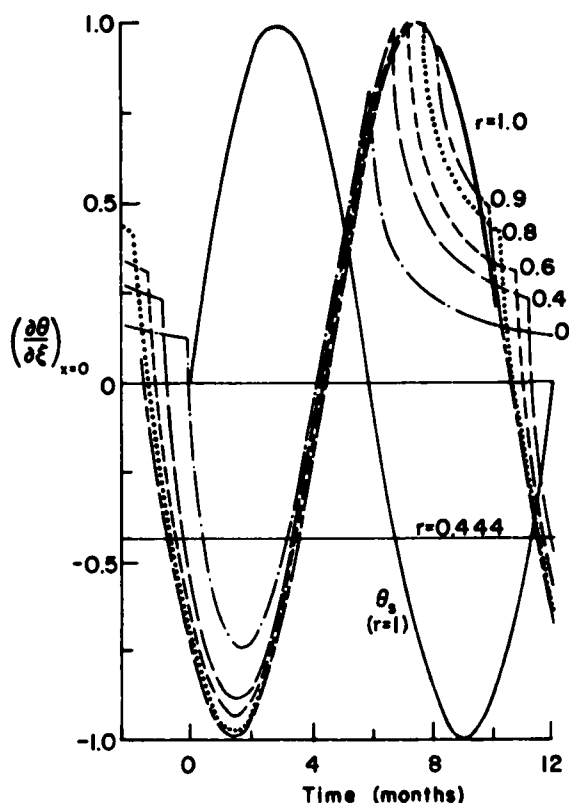


Figure 4. Nondimensional river bottom temperature gradient, evaluated at the sediment/water interface, as a function of time and r . The nondimensional river bottom temperature θ_s is also shown. In the example in the text, $r = 0.444$, and the θ_s curve is truncated at the level shown.

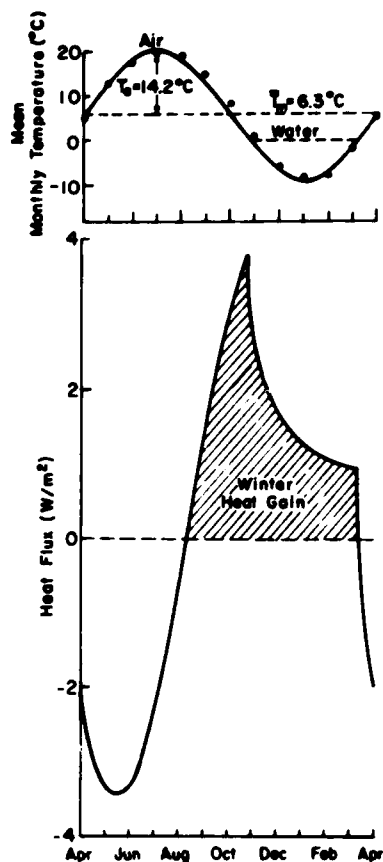


Figure 5. An example using the curves in Figure 4 for Hanover, N.H. Variation of monthly mean temperatures is shown at the top, based on data from nearby weather stations. The corresponding river bottom heat flux response is shown below.

amplitude (T_a) is 14.2°C , from which $r = 6.3/14.2 = 0.444$. Initial time for the sinusoid that fits the air temperature variation is approximately mid-April. Corresponding heat flux response over the months may then be read off Figure 4, with values between the curves for $r = 0.4$ and $r = 0.6$. The transformation to $\partial T/\partial x$ from $\partial\theta/\partial\xi$ is

$$\begin{aligned} \frac{\partial T}{\partial x} &= \frac{T_a}{\sqrt{K/f}} \frac{\partial\theta}{\partial\xi} = \frac{14.2^\circ\text{C}}{(1.31 \times 10^{-7} \text{ m}^2 \text{ s}^{-1} \cdot 86400 \cdot 365 \text{ s})^{1/2}} \frac{\partial\theta}{\partial\xi} \\ &= 7.0 \frac{\partial\theta}{\partial\xi} \end{aligned}$$

The actual heat flux is then found from $k \partial T / \partial x$ where k is the conductivity of the bottom material (taken as the value for water at 0°C ; $k = 0.55 \text{ W m}^{-1}\text{C}^{-1}$). Figure 5 shows heat flux values based on this value of k and on values from Figure 4. Examination of Figure 5 shows that the heat gain to the water reaches a maximum in November (about the time of initial ice cover formation) and then decreases, rapidly at first, over the winter.

This analysis is most applicable to rivers and to reservoirs where sufficient through-flow exists during winter to ensure that the bottom water is at 0°C throughout the winter. In natural lakes with little through-flow, the initial temperature at the onset of ice is often somewhat above 0°C (but seldom above 4°C). In this case a good first approximation is to use the same procedure but with an r value determined using the initial temperature, rather than 0°C . However, since the initial temperature at onset of ice is determined by the particular sequence of meteorological events after the 4°C isothermal state has been achieved, and these events vary from year to year, it is difficult to predict the initial temperature.

As was discussed earlier, a complete solution to the problem would include the effect of elevation of the bottom water temperatures as a result of the heat flux from the bottom. The effect of this warming would be to cause the falling limb of the "winter heat gain" curve (Fig. 5) to fall further, but would have little effect on the higher values of heat flux near the beginning of the winter.

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